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Equation of a line

Learning Objectives Determine the equation of a line given the \(y\) intercept and slope Use a graph to extract the \(y\) intercept and slope of a line to determine its equation Determine the equation of a line given the \(y\) intercept and a point Determine the equation of a line given two points Find the \(y\) intercept of the graph below Find the slope of the line that contains the points (-1, 2) and (3, -6) Solve for \(y\) to get the equation into slope-intercept form: \(y - 3 = -4(x + 1)\). Answer \(b = -2\) \(m = -2\) \(y = -4x-1\) In the last few sections, we've talked about how we can use an equation to look at the graphs of a line or points on a line. In this section, we look at different ways to deduce the equation of a line given information such as slope, intercepts, points, or just a graph. First, let's recall a particularly helpful form for the equation of a line -- the slope-intercept form -- and discuss a strategy we can use to deduce an equation of this form. If the line \(L\) intercepts the y-axis at the point (0, b) and has slope m, then the equation of the line is \[y=m x+b. \label{slopeintercept eq}\] This form of the equation of a line is called the slope-intercept form. The function defined by the equation \[f(x)=m x+b\] is called a linear function. It is important to note two key facts about the slope-intercept form y = mx + b. The coefficient of x (the m in y = mx + b) is the slope of the line. The constant term (the b in y = mx+b) is the y-coordinate of the y-intercept (0, b). So in order to create an equation for a line using this form, all we would need to deduce is those two pieces of information, slope and intercept. When given the slope of a line and the y-intercept of the line, use the slope-intercept form as follows: Substitute the given slope for m in the formula \(y = mx + b\). Substitute the y-coordinate of the y-intercept for b in the formula \(y = mx + b\). For example, if the line has slope −2 and the y-intercept (the point where the line crosses the y-axis) is (0, 3), then substitute m = −2 and b = 3 into Equation \ref{slopeintercept eq} to obtain \[y = −2x + 3.onumber\] As we did in the last chapter, we can easily determine the slope and intercept of a line if the equation is written in slope-intercept form, \(y=mx+b\). Now we will do the reverse—we will start with the slope and y-intercept and use them to find the equation of the line. Find the equation of a line with slope \(−9\) and y-intercept \((0,−4)\). Answer Since we are given the slope and y-intercept of the line, we can substitute the needed values into the slope-intercept form, \(y=mx+b\). Name the slope. Name the y-intercept. Substitute the values into \(y=mx+b\). Find the equation of a line with slope \(25\) and y-intercept \((0,4)\). Answer \(y=\frac{2}{5}x+4\) Find the equation of a line with slope \(−1\) and y-intercept \((0,−3)\). Answer \(y=−x−3\) Sometimes, the slope and intercept need to be determined from the graph. Given the graph of the line in Figure \(\PageIndex{4}\)(a), determine the equation of the line. Solution First, locate the y-intercept of the line, which we've labeled P(0, −1) in Figure \(\PageIndex{4}\)(b). In the formula y = mx + b, recall that b represents the y-coordinate of the y-intercept. Thus, b = −1. Figure \(\PageIndex{3}\). The line has y-intercept at (0, 3) and slope −2/3. Secondly, we need to determine the slope of the line. In Figure \(\PageIndex{4}\)(b), start at the point P, move 2 units to the right, then 3 units upward to the point Q(2, 2). This makes the slope \[m=\frac{\Delta y}{\Delta x}=\frac{3}{2}\] Substitute m = 3/2 and b = −1 into the slope-intercept form y = mx + b to obtain \[y=\frac{3}{2} x-1\] which is the desired equation of the line. Figure \(\PageIndex{4}\). Determining the equation of a line from its graph. Find the equation of the line shown in the graph. Answer We need to find the slope and y-intercept of the line from the graph so we can substitute the needed values into the slope-intercept form, \(y=mx+b\). To find the slope, we choose two points on the graph. The y-intercept is \((0,−4)\) and the graph passes through \((3,−2)\). Find the slope, by counting the rise and run. Find the y-intercept. Substitute the values into y=mx+b.y=mx+b. Find the equation of the line shown in the graph. Answer \(y=\frac{3}{5}x+1\) Find the equation of the line shown in the graph. Answer \(y=\frac{4}{3}x−5\) Finding an equation of a line using the slope-intercept form of the equation works well when you are given the slope and y-intercept or when you read them off a graph. But what happens when you have another point instead of the y-intercept? We are going to use the slope formula to derive another form of an equation of the line. Suppose we have a line that has slope m and that contains some specific point \((x_0,y_0)\) and some other point, which we will just call \((x,y)\). We can write the slope of this line and then change it to a different form. \(\begin{array} {llll} {} &{m} &= &{\frac{y-y_0}{x-x_0}} \\ {\text{Multiply both sides of the equation by }x−x_1.} &{m(x-x_1)} &= &{\left(\frac{y−y_1}{x−x_1} \right)(x−x_1)} \\ {\text{Simplify.}} &{m(x-x_1)} &= &{y-y_1} \\ {\text{Rewrite the equation with theyterms on the left.}} &{y-y_1} &= &{m(x-x_1)} \\ \end{array} \) This format is called the point-slope form of an equation of a line. If line L passes through the point \(\left(x_{1}, y_{1}\right)\) and has slope m, then the equation of the line is \[y-y_{1}=m\left(x-x_{1}\right)\] This form of the equation of a line is called the point-slope form. To use the point-slope form of a line, follow these steps. When given the slope of a line and a point on the line, use the point-slope form as follows: Substitute the given slope for m in the formula \(y-y_{1}=m\left(x-x_{1}\right)\). Substitute the coordinates of the given point for x0 and y0 in the formula \(y-y_{1}=m\left(x-x_{1}\right)\). For example, if the line has slope −2 and passes through the point (3, 4), then substitute \(m=-2, x_{1}=3,\) and \(y_{1}=4\) in the formula \(y-y_{1}=m\left(x-x_{1}\right)\) to obtain \[y-4=-2(x-3)onumber\]. It is common to rewrite this equation in slope-intercept form, because it's such a popular format. To do this, we just get \(y\) by itself on the left and then simplify everything on the left: \[y-4=-2(x-3)onumber\] \[y=-2(x-3)+4onumber\] \[y=-2x+6+4onumber\] \[y=-2x+10onumber.\] We can use the point-slope form of an equation to find an equation of a line when we know the slope and at least one point, even if it's not the \(y\) intercept. Then, we will rewrite the equation in slope-intercept form. Most applications of linear equations use the the slope-intercept form. How to Find an Equation of a Line Given a Point and the Slope Find an equation of a line with slope \(m=−\frac{1}{3}\) that contains the point \((6,−4)\). Write the equation in slope-intercept form. Answer Find the equation of a line with slope \(m=−\frac{2}{5}\) and containing the point \((10,−5)\). Answer \(y=−\frac{2}{5}x−1\) Find the equation of a line with slope \(m=−\frac{3}{4}\), and containing the point \((4,−7)\). Answer \(y=−\frac{3}{4}x−4\) Let's summarize the steps more succinctly for easy reference: Procedure for Using the Point-Slope Form of a Line to Find an Equation of a Line Using a Slope and a Point Identify the slope. Identify the point. Substitute the values into the point-slope form, \(y−y_1=m(x−x_1)\). Write the equation in slope-intercept form. Find an equation of a horizontal line that contains the point \((−2,−6)\). Write the equation in slope-intercept form. Answer Every horizontal line has slope 0. We can substitute the slope and points into the point-slope form, \(y−y_1=m(x−x1)\). Identify the slope. Identify the point. Substitute the values into y−y1=m(x−x1).y−y1=m(x−x1). Simplify. Write in slope-intercept form. It is in y-form, but could be written \(y=0x−6\). Did we end up with the form of a horizontal line, \(y=a\)? Find the equation of a horizontal line containing the point \((−3,8)\). Answer \(y=8\) Find the equation of a horizontal line containing the point \((−1,4)\). Answer \(y=4\) In the next example we'll see how to find an equation of a line when just two points are given. So far, we have two options for finding an equation of a line: slope-intercept or point-slope. When we start with two points, it makes more sense to use the point-slope form. But then we need the slope. Can we find the slope with just two points? Yes. Then, once we have the slope, we can use it and one of the given points to find the equation. How to Find the Equation of a Line Given Two Points Find an equation of a line that contains the points \((−3,−1)\) and \((2,−2)\) Write the equation in slope-intercept form. Answer Find the equation of a line containing the points \((−2,−4)\) and \((1,−3)\). Answer \(y=\frac{1}{3}x−\frac{10{3}\) Find the equation of a line containing the points \((−4,−3)\) and \((1,−5)\). Answer \(y=−\frac{2}{5}x−\frac{23}{5}\) The steps are summarized here. TO FIND AN EQUATION OF A LINE GIVEN TWO POINTS. Find the slope using the given points. \(m=\frac{y_2−y_1}{x_2−x_1}\) Choose one point. Substitute the values into the point-slope form: \(y−y_1=m(x−x_1)\). Write the equation in slope-intercept form. Find an equation of a line that contains the points \((−3,5)\) and \((−3,4)\). Write the equation in slope-intercept form. Answer Again, the first step will be to find the slope. Find the slope of the line through\((−3,5)\)and\((−3,4)\). \[m=\frac{y_2−y_1}{x_2−x_1} onumber\] \[m=\frac{4−5}{−3−(−3)} onumber\] \[m=\frac{−1}{0} onumber\] The slope is undefined. This tells us it is a vertical line. Both of our points have an x-coordinate of \(−2\). So our equation of the line is \(x=−2\). Since there is no y, we cannot write it in slope-intercept form. You may want to sketch a graph using the two given points. Does your graph agree with our conclusion that this is a vertical line? Find the equation of a line containing the points \((5,1)\) and \((5,−4)\). Answer \(x=5\) Find the equation of a line containing the points \((−4,4)\) and \((−4,3)\). Answer \(x=−4\) We have seen that we can use either the slope-intercept form or the point-slope form to find an equation of a line. Which form we use will depend on the information we are given. To Write an Equation of a Line If given: Use: Form: Slope and y-intercept slope-intercept \(y=mx+b\) A graph slope-intercept \(y=mx+b\) Slope and a point point-slope \(y−y_1=m(x−x_1)\) Two points point-slope \(y−y_1=m(x−x_1)\) Key Concepts There are two useful forms for an equation -- the slope-intercept form and the point-slope form The point-slope form is often the easiest to use to create the equation initially The slope-intercept form is the most popular final form The slope-intercept form of a line is \(y = mx + b\), and only requires the slope \(m\) and the \(y\) intercept \(b\) to create it When given the slope of a line and the y-intercept of the line, use the slope-intercept form as follows: Substitute the given slope for m in the formula \(y = mx + b\). Substitute the y-coordinate of the y-intercept for b in the formula \(y = mx + b\). When we know a point \((x_1, y_1)\) that is on the line, the point-slope form of a line is \(y - y_1 = m(x - x_1)\); but it is commonly rewritten to be in slope-intercept form When given the slope of a line and a point on the line that may not be the \(y\) intercept, use the point-slope form as follows: Identify the slope. Identify the point. Substitute the values into the point-slope form, \(y−y_1=m(x−x_1)\). Write the equation in slope-intercept form. When given two points, \((x_1, y_1)\) and \((x_2, y_2)\), the point-slope form is the easiest to use; we just need to do extra work to get the slope first Use the point-slope form as follows: Find the slope using the given points. \(m=\frac{y_2−y_1}{x_2−x_1}\) Choose one point. Substitute the values into the point-slope form: \(y−y_1=m(x−x_1)\). Write the equation in slope-intercept form. Line is an endless line, which forms the shortest path between any of two its points. Any equation of a line on plane can by written in the general form A x + B y + C = 0 where A and B are not both equal to zero. The general equation of a line when B ≠ 0 can be reduced to the next form y = k x + b where k is the slope of the line and b is the y-intercept. Slope of the line is equal to the tangent of the angle between this line and the positive direction of the x-axis. The y-coordinate is the location where line crosses the y-axis. k = tg φ If the line passes through two points A(x1, y1) and B(x2, y2), such that x1 ≠ x2 and y1 ≠ y2, then equation of line can be found using the following formula x - x1 = y - y1 x2 - x1 y2 - y1 Parametric equation of the line can be written as x = l t + x0 y = m t + y0 where N(x0, y0) is coordinates of a point that lying on a line, a = {l, m} is coordinates of the direction vector of line. If you know the coordinates of the point A(x0, y0) If you know the coordinates of the point n = {l; m}, then the equation of the line can be written in the canonical form using the following formula Example 1. Find the equation of a line passing through two points A(1, 7) and B(2, 3). Solution. We use the formula for the equation of a straight line passing through two points x - 12 - 1 = y - 73 - 7 From this equation, we express y in terms of x x - 11 = y - 7-4 Find the Slope intercept form of line equation. Multiply equations by -4. y - 7 = -4(x - 1) y = -4x + 11 Find the Parametric equations of this line We use MN as direction vector of line. MN = {2 - 1; 3 - 7} = {1; -4} We use coordinates of point M in parametric equations of line x = t + 1y = -4t + 7 Example 2. Find the equation of a line passing through two points M(1, 3) and N(2, 3). Solution. It is impossible to use Equation of the line passing through two different points, since My - Ny = 0. Find the Parametric equations of this line. We use MN as direction vector of line. MN = {2 - 1; 3 - 3} = {1; 0} We use coordinates of point M in parametric equations of line x = t + 1y = 3 If the line passes through two points A(x1, y1, z1) and B(x2, y2, z2), such that x1 ≠ x2, y1 ≠ y2 and z1 ≠ z2, then equation of line can be found using the following formula x - x1 = y - y1 = z - z1 x2 - x1 y2 - y1 z2 - z1 Parametric equation of the line can be written as x = l t + x0 y = m t + y0 z = n t + z0 where (x0, y0, z0) is coordinates of a point that lying on a line, {l; m; n} is coordinates of the direction vector of line. If you know the coordinates of the point A(x0, y0, z0) that lies on the line and the direction vector of the line n = {l; m; n}, then the equation of the line can be written in the canonical form using the following formula. x - x0 = y - y0 = z - z0 l m n If the line is the intersection of two planes, then the equation of line can be found as the solution of the following system of equations A1x + B1y + C1z + D1 = 0 A2x + B2y + C2z + D2 = 0 on condition that there has not been equality Move the m and b slider bars to explore the properties of a straight line graph. See: The effect of changes in m The effect of changes in b The effect of a negative value of m The effect of a negative value of b How to create a horizontal line ../algebra/images/graph-straight-line.js Read more about the Equation of a Straight Line Find the Equation from 2 Points Now see how two points can change the line equation. Try to make: y = x y = x + 2 y = −2x + 8 y = 4 x = 4 ../geometry/images/geom-line-equn.js?mode=equn Copyright © 2025 Rod Pierce home / algebra / linear equations / equation of a lineThe equation of a straight line is commonly written as y = mx + b where m is the slope and b is the y-intercept. The slope determines how steep the line is. The y-intercept tells us the point at which the line intersects the y-axis. The x and y tell us the position relative to the x- and y-axes respectively. Thus, given any 2 points on the line, or 1 point and the slope, we can graph the line. How to find the slope The slope of a line is often described as "rise over run." It tells us how much the y-value changes as the x-value changes. Given a slope of 2, this means that for every 1 unit change in x, y changes 2 units. Because the slope is positive, the line increases (moves up) as the graph moves from left to right. If the slope were negative, the line would decrease instead. Example Graph the lines y = 2x + 3 and y = -2x + 3 Based on the equation of the line, the y-intercept is 3, meaning that the line goes crosses y-axis at the point (0, 3). The slope is 2, or 2/1, so for every 1 unit of change in y, there is a 2 units change in y. Using the y-intercept as a reference point, we can either move right 1 and up 2, or left 1 and down 2 to determine the next point. Once we have two points, we can just connect the points to graph the line. The figure below shows the two possible points on either side of the chosen reference point (the y-intercept). The only difference between the two equations above is that the slope is positive in one but negative in the other. Below is the graph of the line given that the slope is -2. The equation of a line is an expression that describes a straight line by defining the relationship between its points in a coordinate plane. Each point on the line is represented by a coordinate (x, y). There are an infinite number of points on a line. It is thus a linear equation with a degree of 1. The equation of a line can be expressed in various forms depending on the given information. The standard or general form of the equation of a line: For One Variable is Ax + B = 0 For Two Variable is Ax + By = C Here, A, B, and C are constants x and y are variables The equation 2x + 3y = 6 represents a line in standard form. The equation of a line is most commonly written in the slope-intercept form, which is written as: y = mx + b (written as y = mx + c in UK) Here, m is the slope of the line b is the y-intercept (where the line crosses the y-axis) Thus, we need the slope and the y-intercept to find the equation of the line. The slope can be positive (the line goes up as you move to the right), negative (the line goes down as you move to the right), or zero (a horizontal line). The line is said to be undefined if it is vertical. For example, If a line has a slope of 2 and a y-intercept of -3, its equation is: y = 2x – 3. The equation has a positive slope. Given a point on the line and the slope, we use the point-slope form, which is written as: y – y1 = m(x – x1) Here, m is the slope of the line (x1, y1) is the coordinates of a point For example, If a point is (3, 4) and a slope is 5, the equation of the line is: y – 4 = 5(x – 3) When any two points of a line are known, say (x1, y1) and (x2, y2), we use the two-point form: ${\dfrac{y-y_{1}}{y_{2}-y_{1}}=\dfrac{x-x_{1}}{x_{2}-x_{1}}}$ If (2, 3) and (4, 7) are two given points of a line, then the equation is: ${\dfrac{y-3}{7-3}=\dfrac{x-2}{4-2}}$, which simplifies to y – 3 = 2(x – 2) If the line makes intercepts both at the x- and y-axis, say at a and b, respectively, we use the intercept form: ${\dfrac{x}{a}+\dfrac{y}{b}=1}$ For example, If a line crosses the x-axis at 4 and the y-axis at 2, the equation of the line is: ${\dfrac{x}{4}+\dfrac{y}{2}=1}$ Note: Sometimes, a linear equation can be written as a function using f(x) instead of y. For example, f(x) = 3x – 7, g(x) = x + 1, and h(x) = -5x + 3 However, functions are not always written using f(x). For example, f(y) = 3y – 7, g(p) = p + 1, and h(u) = -5u + 3 y = b is the equation of a horizontal line, where b is the y-coordinate of any point on the line. x = a is the equation of a vertical line, where a is the x-coordinate of any point on the given line. Below are the different ways we learned to express the equation of a straight line: Determine the equation of a line passing through the points (2, 5) and (6, 9).Solution:Using the two-point form, m = ${\dfrac{9-5}{6-2}=\dfrac{4}{4}=1}$Thus, the equation is: y – 5 = 1(x – 2)⇒ y – 5 = x – 2⇒ y = x + 5 – 2⇒ y = x + 3Thus, the equation of the line is y = x + 3 Find the equation of a line that passes through the point (3, -2) with a slope of -4.Solution:Using point-slope form, we gety + 2 = -4(x – 3)⇒ y + 2 = -4x + 12⇒ y = -4x + 12 – 2⇒ y = -4x + 10Thus, the equation of the line is y = -4x + 10 Last modified on March 20th, 2025 The gradient (also called slope) of a line tells us how steep it is. To find the gradient: Divide the vertical change (how far it goes up or down) by the horizontal change (how far it moves sideways). Gradient = Change in YChange in X Have a play (drag the points): geometry/images/geom-line-equn.js?mode=slope The gradient = 3 3 = 1 So the gradient is equal to 1 The gradient = 4 2 = 2 The line is steeper, and so the gradient is larger. The gradient = 3 5 = 0.6 The line is less steep, and so the gradient is smaller. Moving from left-to-right, the cyclist has to : Nosedive on a Negative slope Push up on a Positive slope When measuring the line: Starting from the left and going across to the right is positive (but going across to the left is negative). Up is positive, and down is negative Gradient = −4 2 = −2 That line goes down as you move along, so it has a negative gradient. Gradient = 0 5 = 0 A line that goes straight across (horizontal) has a gradient of zero. Straight Up and Down Gradient = 3 0 = undefined That last one is a bit tricky ... you can't divide by zero, so a "straight up and down" (vertical) line's gradient is "undefined". Rise and Run We also call the horizontal change "run", and the vertical change "rise" or "fall": They are just different words, none of the calculations change. 2068, 2069, 2070, 2071, 2072, 5316, 5317, 5318, 5319, 5320 Copyright © 2025 Rod Pierce The standard forms of the equation of a line are: Slope-intercept form Intercept form Normal form Let us learn all the straight lines formulas along with the general equation of a line and different forms to find the equation of a straight line in detail here. General Equation of a Line The general equation of a line in two variables of the first degree is represented as Ax + By +C = 0, A, B ≠ 0 where A, B and C are constants which belong to real numbers. When we represent the equation geometrically, we always get a straight line. Below is a representation of straight-line formulas in different forms: Slope-intercept Form We know that the equation of a straight line in slope-intercept form is given as: Where m indicates the slope of the line and c is the y-intercept When B ≠ 0 then, the standard equation of first degree Ax + By + C = 0 can be rewritten in slope-intercept form as: y = (− A/B) x − (C/B) Thus, m= –A/B and c = –C/B . Intercept Form The intercept of a line is the point through which the line crosses the x-axis or y-axis. Suppose a line cuts the x-axis and y-axis at (a, 0) and (0, b), respectively. Then, the equation of a line making intercepts equal to a and b on the x-axis and the y-axis respectively is given by: x/a + y/b = 1 Now in case of the general form of the equation of the straight line, i.e. Ax+By+C = 0, if C ≠ 0, then Ax + By + C = 0 can be written as; x/(-C/A) + y/(-C/B) = 1 where a = -C/A and b = – C/B Normal Form The equation of the line whose length of the perpendicular from the origin is p and the angle made by the perpendicular with the positive x-axis is given by α is given by: x cos α+y sin α = p This is known as the normal form of the line. In case of the general form of the line Ax + By + C = 0 can be represented in normal form as: A cos α = B sin α = – p From this we can say that cos α = -p/A and sin α = -p/B. Also it can be inferred that, cos2α + sin2α = (p/A)2 + (p/B)2 1 = p2 (A2 + B2/A2 .B2) From the general equation of a straight line Ax + By + C = 0, we can conclude the following: The slope is given by -A/B, given that B ≠ 0. The x-intercept is given by -C/A and the y-intercept is given by -C/B. It can be seen from the above discussion that: , , If two points (x1, y1) and(x2, y2)are said to lie on the same side of the line Ax + By + C = 0, then the expressions Ax1+ By1 + C and Ax2 + By2 + C will have the same sign or else these points would lie on the opposite sides of the line. Straight Line Formulas Let us accumulate the straight line formulas we have discussed so far: Slope (m) of a non-vertical line passing through the points (x1 , y1) and (x2, y2) m=(y2-y1)/(x2-x1), x1≠x2 Equation of a horizontal line y = a or y=-a Equation of a vertical line x=b or x=-b Equation of the line passing through the points (x1 , y1) and (x2, y2) y-y1= [(y2-y1)/(x2-x1)]×(x-x1) Equation of line with slope m and intercept c y = mx+c Equation of line with slope m makes x-intercept d. y = m (x – d). Intercept form of the equation of a line (x/a)+(y/b)=1 The normal form of the equation of a line x cos α+y sin α = p Example of Straight Lines To understand this concept better go through the below examples: (1) The equation of a line is given by, 2x – 6y +3 = 0. Find the slope and both the intercepts. Solution: The given equation 2x – 6y + 3 = 0 can be represented in slope-intercept form as: y = x/3 + 1/2 Comparing it with y = mx + c, Slope of the line, m = 1/3 Also, the above equation can be re-framed in intercept form as; x/a + y/b = 1 2x – 6y = -3 x/(-3/2) – y/(-1/2) = 1 Thus, x-intercept is given as a = -3/2 and y-intercept as b = 1/2. (2) The equation of a line is given by, 13x – y + 12 = 0. Find the slope and both the intercepts. Solution: The given equation 13x – y + 12 = 0 can be represented in slope-intercept form as: y = 13x + 12 Comparing it with y = mx + c, Slope of the line, m = 13 Also, the above equation can be re-framed in intercept form as; x/a + y/b = 1 13x – y = -12 x/(-12/13) + y/12 = 0 Thus, x-intercept is given as a = -12/13 and y-intercept as b = 12. To learn all the concepts in Math in a more engaging way, register at BYJU'S. Also, watch interesting videos on various maths topics by downloading BYJU'S– The Learning App from Google Play Store or the app store. The general equation of a straight line is y = mx + c, where m is the slope of the line and c is the y-intercept. It is the most common form of the equation of a straight line that is used in geometry. The equation of a straight line can be written in different forms such as point-slope form, slope-intercept form, intercept form, standard form, etc. A straight line is a two-dimensional geometrical entity that extends on both ends till infinity. In this article, we will explore the concept of the equation of a straight line in different forms. Try your hands at solving a few interesting examples and questions for a better understanding of the concept. What is the Equation of Straight Line? The equation of a straight line is an linear equation in two variables (usually x and y) and is satisfied by every point on the line. i.e. it is a mathematical equation that gives the relation between the coordinate points lying on that straight line. It can be written in different forms and tells the slope, x-intercept, and y-intercept of the line. It also can be used to find the points on the line. Mostly, the equation of a straight line is found by using point-slope form, slope-intercept form, two-point form, standard form, etc. Let us go through the formula for the equation of a straight line. The most common formulas to find equation of straight line are mentioned below. Equation of Straight Line Formulas The equation of straight line formula varies depending on what information is available about the line like slope, intercepts, etc. Note that the slope of a line with two points (x1, y1) and (x2, y2) is calculated by the formula m = (y2 - y1)/(x2 - x1). Here are different straight line formulas. We will study each of these in detail in the section below. Forms of Equation of a Straight Line The equation of a straight line usually involves slope. Suppose a line l makes an angle of θ with a positive direction of the x-axis, the angle θ is called the inclination of the line and tan θ is called the slope of the line. Note that x-axis has a slope 0. In fact, all lines parallel to the x-axis have a slope of 0. Also, the slope of all the lines parallel to y-axis including the y-axis is not defined. Now, Let us go through different forms of equations of a straight line. Point-Slope Form The equation of a straight line whose slope is m and which passes through a point (x1, y1) is found using the point-slope form. The equation of the point-slope form is: y - y1 = m (x - x1), where (x, y) is an arbitrary point on the line. Let us see how to find the point-slope form. We will derive this formula using the equation for the slope of a line. Let us consider a line whose slope is m. Let us assume that (x1, y1) is a known point on the line. Let (x, y) be any other random point on the line whose coordinates are not known. We know that the equation for the slope of a line is: Slope = Difference in y-coordinates/Difference in x-coordinates ⇒ m = (y - y1)/(x - x1) Multiplying both sides by (x - x1), m (x - x1) = (y - y1) This can be written as, (y - y1) = m (x - x1) Hence the point-slope form of the equation of a straight line is proved. Two Point Form Consider a line with two points (x1, y1) and (x2, y2) on it. Then its slope can be calculated by the formula m = (y2 - y1)/(x2 - x1). Substituting this in the above point-slope form, we get the two point form as y - y1 = (y2 - y1)/(x2 - x1) (x - x1). Slope-Intercept Form Now, suppose a line is given to you with its slope m and its y-intercept. Say, a line intersects the y-axis at the point (0, c). Using the point-slope form we have y - c = m (x - 0) ⇒ y = mx + c, where c is the y-intercept. This is called the slope-intercept form of a line. Note: If d is the x-intercept, then the slope-intercept form of the equation of the line is y = m(x - d). Intercept Form If (a, 0) and (0, b) are the x and y-intercepts of a line respectively. Then its slope is, m = (b - 0)/(0 - a) = -b /a. Then its equation using the point-slope form is: y - 0 = -b/a (x - a) Multiplying both sides by a ay = -bx + ab bx + ay = ab Dividing both sides by ab, x/a + y/b = 1 Standard Form The standard form of a straight line is given by ax + by = c, where a, b, c are real numbers. We can consider any form of a line into standard form. Let us consider an example to transform the equation y = 2x - 1 in the standard form. Subtract 2x from both sides of the equation, we have y - 2x = 2x - 1 - 2x ⇒ y - 2x = -1 ⇒ 2x - y = 1 So, we obtain the standard form of the equation of the line as 2x - y = 1. Equation of Straight Line on Graph The graph of a linear equation in one variable x forms a vertical line parallel to the y-axis and the graph of the equation of a straight line in one variable y is a horizontal line parallel to the x-axis The graph of a linear equation in two variables x and y forms a straight line with some slope. If a straight line is increasing from left to right, its slope is positive. If it is decreasing, its slope is negative. Important Notes on Equation of Straight Line: The equation of a straight line is also called a linear equation in two variables. If the product of slopes of two straight lines is -1, then lines are perpendicular to each other. If two straight lines are parallel to each other, then they have the same slope. Point Slope Form: (y - y1) = m (x - x1) Slope-Intercept Form: y = mx + c Standard Form = ax + by = c ☛Related Topics: Linear Equation Formula Equation of Staright Line Calculator Cuemath is one of the world's leading math learning platforms that offers LIVE 1-to-1 online math classes for grades K-12. Our mission is to transform the way children learn math, to help them excel in school and competitive exams. Our expert tutors conduct 2 or more live classes per week, at a pace that matches the child's learning needs. Example 1: Find the equation of a straight line that passes through the points (1, 3) and (-2, 4). Write the equation in standard form. Solution: To determine the equation of the line, we will use the formula point-slope form. For this, we first need to find the slope of the line. Slope = (4-3)/(-2-1) = -1/3 Therefore, the equation of the line passing through (1, 3) and (-2, 4) is y - 4 = (-1/3) (x + 2) ⇒ y - 4 = -x/3 - 2/3 ⇒ y + x/3 = 4 - 2/3 ⇒ x + 3y = 10 Answer: The required equation is x + 3y = 10. Example 2: The cost of a notebook is $5 more than twice the cost of a pen. Represent the situation as an equation of a straight line. Solution: Assume the cost of pen = $x and the cost of notebook = $y. Then, according to the question, we have y = 2x + 5 which is the equation of a straight line. Answer: y = 2x + 5 Example 3: The equation of a straight line is given by 3x - 4y = 12. Convert this into the intercept form and hence find the intercepts. Solution: The equation of given line is: 3x - 4y = 12 Dividing both sides by 12, 3x/12 - 4y/12 = 12/12 x/4 + y/(-3) = 1 This is in the intercept form x/a + y/b = 1. Hence, the intercepts are (a, 0) = (4, 0) and (0, b)= (0, -3). Answer: x/4 + y/(-3) = 1; Intercepts: (4, 0) and (0, -3). View Answer > go to slidego to slidego to slide Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our certified experts Book a Free Trial Class FAQs on Equation of Straight Line The equation of a straight line is a linear equation in x and y that gives the relation between the coordinate points that lie on that line. The equation of a straight line is usually of the form y = mx + c, where m is the slope of the line and c is its y-intercept. What is the Formula for the Equation of a Straight Line? The equation of a straight line can be found using different formulas: Point-slope form: y - y1 = m (x - x1) Two-point form: y - y1 = [(y2-y1) / (x2-x1)] (x - x1) Slope-intercept form: y = mx + c Intercept form: x/a + y/b = 1 General form: ax + by = c Normal form: x cos θ + y sin θ = p How to Find the Equation of a Straight Line? The equation of any straight line can be found by using the point-slope form y - y1 = m (x - x1), where m = slope of the line and it can be calculated by the formula tan θ (or) (y2 - y1) / (x2 - x1). (x1, y1) is a point on the line. How to Convert an Equation of Straight Line From Point-Slope Form to Slope-Intercept Form? An equation of straight line can be converted from point-slope form to the slope-intercept form just by simplifying it such that the left-side of the equation has only y and all the other terms are moved to the right side. For example, y - 2 = 3 (x - 1) is in the point-slope form. If we simplify it, y - 2 = 3x - 3, adding 2 on both sides, y = 3x - 1, which is in the slope-intercept form. How Do you Find the Slope and Y-intercept of an Equation of a Straight Line? By converting an equation of straight line into slope-intercept form y = mx + c, we can find its slope m and the y-intercept c. For example, if the equation is 2x - 3y = 1, to find its slope and y-intercept, we first need to solve it for y. Then we get y = (2/3)x - 1/3. Comparing this with y = mx + c, we get slope, m = 2/3 and y-intercept = (0, c) = (0, -1/3). How Do you Write an Equation for a Vertical and Horizontal Line? The equation of a horizontal line passing through (a, b) is of the form y = b. The equation of a vertical line passing through (a, b) is of the form x = a. How To Find the Equation of a Straight Line when Given Two Points? The equation of a straight line joining two points (x1, y1) and (x2, y2) is given by two-point form y - y1 = (x - x1)[(y2 - y1)/(x2 - x1)]. Alternatively, we can first find its slope using m = (y2 - y1)/(x2 - x1) and then use the point-slope form y - y1 = m (x - x1). How to Graph a Straight Line Using its Equation? To graph a straight line, find any two points on it using its equation. For example: consider the equation y = 3x + 2. Here, we assume any two random numbers for x and find the corresponding y-values using the equation. x y 0 y = 3(0) + 2 = 2 1 y = 3(1) + 2 = 5 Hence, (0, 2) and (1, 5) are two points on the given line. Just plotting them and joining them by a line gives its graph. When do We Use the Normal Form to Find Equation of a Straight Line? When a the normal from the origin to a line makes an angle θ with the positive direction of the x-axis and its perpendicular distance from the origin is p, then its equation can be found only by using normal form which states x cos θ + y sin θ = p. The "point-slope" form of the equation of a straight line is: The equation is useful when we know: one point on the line: (x1, y1) and the slope of the line: m, and want to find other points on the line. Have a play with it (move the point, try different slopes): ../geometry/images/geom-line-equn.js?mode=pt Now let's discover more. What does it stand for? (x1, y1) is a known point m is the slope of the line (x, y) is any other point on the line It is based on the slope: Slope m = change in y change in x = y − y1 x − x1 Starting with the slope: we rearrange it like this: to get this: So, it is just the slope formula in a different way! Now let us see how to use it. slope "m" = 31 = 3 y − y1 = m(x − x1) We know m, and also know that (x1, y1) = (3, 2), and so we have: That is a perfectly good answer, but we can simplify it a little: y − 2 = 3x − 9 y = 3x − 9 + 2 y = 3x − 7 m = −3 1 = −3 y − y1 = m(x − x1) We can pick any point for (x1, y1), so let's choose (0,0), and we have: y − 0 = −3(x − 0) Which can be simplified to: What is the equation for a vertical line? The slope is undefined! In fact, this is a special case, and we use a different equation, like this: Every point on the line has x coordinate 1.5, that's why its equation is x = 1.5 What About y = mx + b ? You may already be familiar with the y=mx+b form (called the slope-intercept form of the equation of a line). It is the same equation, in a different form! The "b" value (called the y-intercept) is where the line crosses the y-axis. So point (x1, y1) is actually (0, b) and the equation becomes: Start with y − y1 = m(x − x1) (x1, y1) is (0, b):y − b = m(x − 0) Which is:y − b = mx Put b on other side:y = mx + b 519, 521, 522, 1160, 1161, 1162, 2074, 2075, 9027, 9028, 9029 Copyright © 2025 Rod Pierce The equation of a straight line is usually written this way: (or "y = mx + c" in the UK see below) Slope or Gradient y value when x=0 (see Y Intercept) y = how far up x = how far along m = Slope or Gradient (how steep the line is) b = value of y when x=0 How do we find "m" and "b"? b is easy: just see where the line crosses the Y axis m (the Slope) needs some calculation: m = Change in Y Change in X Knowing this we can work out the equation of a straight line: m = 2 1 = 2 b = 1 (value of y when x=0) Putting that into y = mx + b gets us: y = 2x + 1 With that equation we can now ... ... choose any value for x and find the matching value for y For example, when x is 1: y = 2×1 + 1 = 3 So when x=1 we have y=3 Check for yourself that x=1 and y=3 is actually on the line. Or we could choose another value for x, such as 7: y = 2×7 + 1 = 15 So when x=7 we have y=15 Positive or Negative Slope? Going from left-to-right, the cyclist has to Push on a Positive Slope: m = −3 1 = −3 b = 0 This gives us: y = −3x + 0 We do not need the zero! So: y = −3x What is the equation for a vertical line? The slope is undefined ... and where does it cross the Y-Axis? In fact, this is a special case, and we use a different equation, not "y=...", but instead we use "x=...". Like this: x = 1.5 Every point on the line has x coordinate 1.5, that is why its equation is x = 1.5 Rise and Run Sometimes the words "rise" and "run" are used. Rise is how far up Run is how far along And so the slope "m" is: m = rise run You might find that easier to remember. Now Play With The Graph ! You can see the effect of different values of m (the slope) and b (the y intercept) at Explore the Straight Line Graph We have been looking at the "slope-intercept" form. The equation of a straight line can be written in many other ways. Another popular form is the Point-Slope Equation of a Straight Line. 358,359,517,518, 1156, 1157, 3204, 3205, 3206, 3207 Different Countries teach different "notation" (as sent to me by kind readers): In the US, Australia, Canada, Eritrea, Iran, Mexico, Portugal, Philippines and Saudi Arabia the notation is: y = mx + b In the UK, Australia (also), Bahamas, Bangladesh, Belgium, Brunei, Bulgaria, Cyprus, Egypt, Germany, Ghana, India, Indonesia, Ireland, Jamaica, Kenya, Kuwait, Malaysia, Malawi, Malta, Nepal, New Zealand, Nigeria, Oman, Pakistan, Peru, Singapore, Solomon Islands, South Africa, Sri Lanka, Turkey, UAE, Zambia and Zimbabwe y = mx + c In Afghanistan, Albania, Algeria, Brazil, China, Czech Republic, Denmark, Ethiopia, France, Lebanon, Netherlands, Kosovo, Kyrgyzstan, Norway, Poland, Romania, South Korea, Surinam, Spain, Tunisia and Viet Nam: y = ax + b In Azerbaijan, China, Finland, Russia and Ukraine: y = kx + b In Greece: ψ = αχ + β In Italy: y = mx + q In Japan: y = mx + d In Cuba and Israel: y = mx + n In Romania: y = gA + C In Latvia and Sweden: y = kx + m In Serbia and Slovenia: y = kx + n In your country: let us know! ... but it all means the same thing, just different letters. 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