Continue

Posted byWilliam SmithFebruary 1, 2024Posted inBlog When we calculate average rate of change of a function over a given interval, we're calculating the average number of units that the function moves up or down, per unit along the ???x???-axis. We could also say that we're measuring how much change occurs in our function's value per unit on the ???x???-axis.How do we find the average rate of change? Given the function and the interval we're interested in (???f(x)??? and ???[x_1,x_2]??? respectively), our first step is to calculate the value of our function at both ends of the interval. Then we plug those values and the ends of the interval into our formula to find average rate of change.The formula for average rate of change is???\frac{\Delta{f}}{\Delta{x}}=\frac{f(x_2)-f(x_1)}{x_2-x_1}???over the interval ???[x_1,x_2]???. ExampleFind the average rate of change over the interval ???[0,4]???.???f(x)=2x^2-2???We'll use the formula for average rate of change:???\frac{\Delta{f}}{\Delta{x}}=\frac{f(x_2)-f(x_1)}{x_2-x_1}???We already know that ???x_1=0??? and that ???x_2=4???. We'll find ???f(x_1)??? and ???f(x_2)??? by plugging ???0??? and ???4??? into the function we've been given, ???f(x)=2x^2-2???.???f(0)??? is???f(0)=2(0)^2-2??????f(0)=-2??????f(4)??? is???f(4)=2(4)^2-2??????f(4)=2(16)-2??????f(4)=30??? When we calculate average rate of change of a function over a given interval, we're calculating the average number of units that the function moves up or down, per unit along the x-axis. Plugging these values into the formula for average rate of change, we get???\frac{\Delta{f}}{\Delta{x}}=\frac{f(x_2)-f(x_1)}{x_2-x_1}??????\frac{\Delta{f}}{\Delta{x}}=\frac{f(4)-f(0)}{4-0}??????\frac{\Delta{f}}{\Delta{x}}=\frac{30-(-2)}{4}??????\frac{\Delta{f}}{\Delta{x}}=\frac{32}{4}??????\frac{\Delta{f}}{\Delta{x}}=8???The average rate of change of the function on ???[0,4]??? is ???8???. This article explores the concept of the average rate of change over an interval, aiming to illuminate this mathematical tool in a manner accessible to everyone.Defining Average Rate of Change Over an IntervalThe average rate of change over an interval refers to the change in the value of a function between two points divided by the difference in the independent variables of these two points. In simpler terms, it measures how much the output (or dependent variable) changes per unit change in the input (or independent variable) over a specific interval.Mathematically, it can be expressed as:Average Rate of Change = [f(b) – f(a)] / (b – a)where f(b) and f(a) are the function values at points b and a, respectively, and b and a are the endpoints of the interval on which the rate of change is being determined. This is essentially the slope of the secant line passing through the points (a, f(a)) and (b, f(b)) on the graph of the function.Figure-1.The average rate of change is fundamental in calculus and underpins more complex ideas, such as the instantaneous rate of change and the derivative.PropertiesMuch like many mathematical concepts, the average rate of change has certain properties integral to its understanding and application. These properties are fundamental aspects of the average rate of change behavior. Here are some of them in detail:LinearityOne of the key properties of the average rate of change is its linearity, which stems from the fact that it represents the slope of the secant line between two points on a function graph. This essentially means that if the function being considered is linear (i.e., it represents a straight line), the average rate of change over any interval is constant and equals the slope of the line.Dependence on IntervalThe average rate of change is dependent on the specific interval chosen. In other words, the average rate of change between two different pairs of points (i.e., different intervals) on the same function can be different. This is particularly evident in non-linear functions, where the average rate of change is not constant.SymmetryThe average rate of change is symmetric in that reversing the interval will only change the sign of the rate. If the average rate of change from 'a' to 'b' is calculated to be 'r,' then the average rate of change from 'b' to 'a' will be '-r.'Interval Average vs. Instantaneous ChangeThe average rate of change over an interval gives an overall view of the behavior of a function within that interval. It does not reflect instantaneous changes within the interval, which may differ greatly. This fundamental concept leads to the idea of a derivative in calculus, which represents the instantaneous rate of change at a point.Connection to Area Under CurveIn the context of integral calculus, the average rate of change of a function over an interval is equal to the average value of its derivative over that interval. This is a consequence of the fundamental theorem of calculus.Exercise Example 1Linear Function ExampleGiven f(x) = 3x + 2. Find the average rate of change from x = 1 to x = 4.SolutionAverage Rate of Change = [f(4) – f(1)] / (4 – 1)Average Rate of Change = [(34 + 2) – (31 + 2)] / (4 – 1)Average Rate of Change = (14 – 5) / 3Average Rate of Change = 3This means that for every unit increase in x, the function increases by 3 units on average between x = 1 and x = 4.Example 2Quadratic Function ExampleSuppose f(x) = x^2. Find the average rate of change from x = 2 to x = 5.Figure-2.SolutionAverage Rate of Change = [f(5) – f(2)] / (5 – 2)Average Rate of Change = [(5^2) – (2^2)] / (5 – 2)Average Rate of Change = (25 – 4) / 3Average Rate of Change = 7Example 3Exponential Function ExampleSuppose f(x) = 2^x. Find the average rate of change from x = 1 to x = 3.Average Rate of Change = [f(3) – f(1)] / (3 – 1)Average Rate of Change = [(2^3) – (2^1)] / (3 – 1)Average Rate of Change = (8 – 2) / 2Average Rate of Change = 3Example 4Cubic Function ExampleSuppose f(x) = x^3. Find the average rate of change from x = 1 to x = 2.Figure-3.SolutionAverage Rate of Change = [f(2) – f(1)] / (2 – 1)Average Rate of Change = [(2^3) – (1^3)] / (2 – 1)Average Rate of Change = (8 – 1) / 1Average Rate of Change = 7Example 5Square Root Function ExampleSuppose f(x) = √x. Find the average rate of change from x = 4 to x = 9.SolutionAverage Rate of Change = [f(9) – f(4)] / (9 – 4)Average Rate of Change = [(√9) – (√4)] / (9 – 4)Average Rate of Change = (3 – 2) / 5Average Rate of Change = 0.2Example 6Inverse Function ExampleSuppose f(x) = 1/x. Find the average rate of change from x = 1 to x = 2.Figure-4.SolutionAverage Rate of Change = [f(2) – f(1)] / (2 – 1)Average Rate of Change = [(1/2) – (1/1)] / (2 – 1)Average Rate of Change = (-0.5) / 1Average Rate of Change = -0.5Example 7Absolute Value Function ExampleSuppose f(x) = |x|. Find the average rate of change from x = -2 to x = 2.SolutionAverage Rate of Change = [f(2) – f(-2)] / (2 – -2)Average Rate of Change = [(2) – (2)] / (2 – -2)Average Rate of Change = 0 / 4Average Rate of Change = 0Example 8Trigonometric Function ExampleSuppose f(x) = sin(x). Find the average rate of change from x = π/6 to x = π/3. (Note that we use radians for x in trigonometric functions.)SolutionAverage Rate of Change = [f(π/3) – f(π/6)] / (π/3 – π/6)Average Rate of Change = [sin(π/3) – sin(π/6)] / (π/6)Average Rate of Change = [(√3/2) – (1/2)] / (π/6)Average Rate of Change = (√3 – 1) / (π/2)Average Rate of Change ≈ 0.577Applications The average rate of change over an interval is widely applicable in various fields. Here are a few examples:PhysicsIn physics, the average rate of change is commonly used in kinematics, the study of motion. For example, the average velocity of an object over a given time interval is the average rate of change of its position with respect to time during that interval. Similarly, the average acceleration is the average rate of change of velocity.EconomicsIn economics and finance, the average rate of change can be used to understand changes in various metrics over time. For instance, it can be used to analyze the average growth rate of a company's revenue or profits over several years. It could also be used to evaluate changes in stock prices, GDP, unemployment rates, etc.BiologyIn population biology and ecology, the average rate of change can be used to measure the growth rate of a population. This could be the rate of change of the number of individuals in a population or the change in the concentration of a substance in an ecosystem.ChemistryIn chemistry, the rate of reaction is essentially an average rate of change—it represents the change in concentration of a reactant or product per unit of time.Environment ScienceIn environmental studies, the average rate of change can be used to measure pollution levels, temperature changes (global warming), deforestation rates, and many more.Medical ScienceIn medical science, it can measure the rate of change in a patient's condition over time. This could be the change in heart rate, blood sugar levels, or tumor growth rate.GeographyIn geography, it's used to assess changes in various parameters over time, such as the erosion rate of a riverbank, glacier melting rates, or even urban sprawl rates.Computer ScienceIn computer science, the average rate of change can be used in algorithms to predict future trends based on past data.These are just a few examples. The average rate of change is an essential mathematical tool that finds wide-ranging applications across virtually all fields of science, technology, and beyond.All images were created with GeoGebra and MATLAB. Julia ŻuławińskaBogna Szyk and Jack Bowater536 people find this calculator helpfulThe average rate of change calculator is here to help you understand the simple concept hidden behind a long, little bit confusing name. What is the rate of change? Generally speaking, it shows the relationship between two factors. Look for a more precise average rate of change definition below. We will also demonstrate and explain the average rate of change formula with a couple of examples of how to use it. Prefer watching over reading? Learn all you need in 90 seconds with this video we made for you: Watch this on YouTube Everything keeps moving. Change is inevitable. Starting with the acceleration of your bike or car, through to population growth, from the blood flow in your veins to the symbiosis of your cells, the rate of change allows us to establish the value associated with those changes. The average rate of change is a rate that describes how one number changes, on average, in relation to another. If you have a function, it is the slope of the line drawn between two points. But don't confuse it with slope. You can use the average rate of change for any given function, not only linear ones. If you want to learn more about slope go to the slope calculator.In the following picture, we marked two points to help you better understand how to find the average rate of change.The average rate of change formula is: A = [f(x_2) − f(x_1)] / [x_2 − x_1] where: (x_1, f(x_1)) – Coordinates of the first point; and (x_2, f(x_2)) – Coordinates of the second point. If it's positive, it means that one coordinate increases as the other also increases. For example, the more you ride a bike, the more calories you burn. It's equal to zero when one coordinate changes but the other one does not. A good example might be not studying for your exams. As time starts running out, the amount of things to learn doesn't change. The average rate of change is negative when one coordinate increases while the other one decreases. Let's say you're going on a vacation. The more time you spend on your travel, the closer you are to your destination.Let's calculate the average rate of change of distance (average speed) of a train going from Paris to Rome (1420.6 km). On the following chart, you can see the change in distance over time:As you see, the speed wasn't constant. The train stopped two times, and in between stops, it went significantly slower. But for calculating the average speed, the only variables that matter are the change in distance and the change in time. So, if the coordinates of the first point are (0, 0), and the coordinates of the second point are the distance between two cities, and the time of travel is (1420.6, 12.5), then: A = (1420.6 - 0) / (12.5 - 0) = 113.648 [km/h] On average, the train was going 113.648 kilometers per hour. Now, let's look at a more mathematical example.You have been given a function: f(x) = x^2 + 5x − 7 Find the average rate of change over the interval [-4, 6]. Find values of your function for both points: f(x_1) = f(-4) = $(-4)^2$ + 5 × (-4) − 7 = -11 f(x_2) = f(6) = 6^2 + 5 × 6 − 7 = 59 Use the average rate of change equation: A = [f(x_2) − f(x_1)] / [x_2 − x_1] = [f(6) − f(-4)] / [6 − (-4)] = [59 − (-11)] / [6 − (-4)] = 70 / 10 = 7 If you enjoyed the average rate of change calculator, feel free to check out our other tools like this distance calculator, where you can find the distance between points or lines.FAQsNot precisely. The average rate of change reflects how a function changes on average between two points. On the other hand, we define the slope of a function as the slope of the line tangent to the curve at a specific point. In a linear function, every point changes identically, so the average rate of change and slope are equal.To find the average rate of change of a function, follow these steps: Get the (x, y) coordinates of the starting point. We'll call these (x_0, y_0). Get the (x, y) coordinates of the endpoint. These will be (x_1, y_1). Replace both within the average rate of change (A) formula: A = (y_1 − y_0)/(x_1 − x_0). The average rate of change of y = 2x is 2. Since it is a linear function, the average rate of change is just the function's slope. In this case, for every change in the x-coordinate, the y-coordinate will double it.If the speed is constant, yes. Speed reflects how the position changes instantaneously with respect to time. So, if an object were moving at a constant speed, the average rate of change in the position would tell us at which speed it is traveling.Check out 46 similar coordinate geometry calculators The average rate of change describes the average rate at which one quantity is changing with respect to another. It gives an idea of how much the function changed per unit in the given interval. In simple terms, in the rate of change, the amount of change in one item is divided by the corresponding amount of change in another. Let's look into the average rate of change formula in detail. What is the Average Rate of Change? The average rate of change of a function f(x) over an interval [a, b] is defined as the ratio of "change in the function values" to the "change in the endpoints of the interval". i.e., the average rate of change can be calculated using [f(b) - f(a)] / (b - a). In other words, the average rate of change (which is denoted by A(x)) is the "ratio of change in outputs to change in inputs". i.e., A(x) = (change in outputs) / (change in inputs) = Δy / Δx = [f(b) - f(a)] / (b - a) Here, Δy is the change in y-values (or) change in the function values and Δx is the change in x-values (or) the change in the endpoints of the interval. Some examples of the average rate of change are: A bus travels at a speed of 80 km per hour. The number of fish in a lake increases at the rate of 100 per week. The current in an electrical circuit decreases 0.2 amperes for a decrease of 1-volt voltage. Average Rate of Change Formula The average rate of change function describes the average rate at which one quantity is changing with respect to something another quantity. The average rate of change formula is given as, A(x) = [f(b) - f(a)] / (b - a) where, A(x) = Average rate of change f(a) = Value of function f(x) at a f(b) = Value of function f(x) at b Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our certified experts Book a free Trial Class Examples Using Average Rate of Change Formula Example 1: Calculate the average rate of change of a function, f(x) = 2x + 10 as x changes from 3 to 7. Solution: Given: f(x) = 2x + 10, a = 3, b = 7. f(3) = 2(3) + 10 = 6 + 10 = 16 f(7) = 2(7) + 10 = 14 + 10 = 24 Using the average rate of change formula, A(x) = [f(b)−f(a)] / (b−a) A(x) = [f(7)−f(3)]/ (7−3) A(x) = (24 - 16) / 4 A(x) = 8/4 A(x) = 2 Therefore, the rate of change is 2. Example 2: Evaluate the average rate of change of the function f(x) = x2 – 5x in the interval 4 ≤ x ≤ 8. Solution: Given: f(x) = x2 – 5x, a = 4, b = 8. f(4) = f(4) = (4)2 – 5(4) = 16 – 20 = -4 f(8) = f(8) = (8)2 – 5(8) = 64 – 40 = 24 Using the average rate of change formula, A(x) = [f(b)−f(a)] / (b−a) = (24−(−4)) / (8−4) = 28/4 = 7 Therefore, rate of change A(x) = 7. Example 3: Find the rate of change of a function, f(x) = 25x + 18 as x changes from 5 to 8 Solution: Given, f(x) = 25x + 18, a = 5 and b = 8 f(5) = 25×5 + 18 = 143 f(8) = 25×8 + 18 = 218 Using the average rate of change formula, A(x) = [f(b)−f(a)] / (b−a) = [218 - 143] / (8 - 5) = 75 / 3 = 25 Therefore, the average rate of change A(x) = 25 The average rate of change is the change one quantity with respect to the change in another. It is a measure of how much the function changed per unit in a particular interval. If f(x) is the function and [a, b] is the interval, then the formula is A(x) = [f(b) - f(a)] / (b - a) What is the Formula to Find the Average Rate of Change? The average rate of change formula is given as, A(x) = [f(b) - f(a)] / (b - a) where, A(x) = Average rate of change f(a) = Value of function f(x) at a f(b) = Value of function f(x) at b How to Find Average Rate of Change Over an Interval? To find the average rate of change of a function f(x) over an interval [a, b]: Find f(a) and f(b). Substitute the values in the formula [f(b) - f(a)] / (b - a) and simplify. Note that it can be calculated using the formula [f(a) - f(b)] / (a - b) as well. But make sure to follow the same order both in the numerator and the denominator. What is the Formula to Find the Rate of Change of a Linear Function? For a linear function, the rate of change is represented by the parameter (m) in the slope-intercept form for a line: y=mx+b, and is visible in a table or on a graph. Is the Average Rate of Change the Same as Slope? The slope is considered as the average rate of change of a point where the average is taken and is reduced to zero. The slope is the rise over the run which is defined as the average rate of change in y coordinates over the change in x coordinates. Download Article Easy instructions for finding the average rate of change Download Article The average rate of change is the slope of a line that intersects two points on a curve. Finding it is a classic pre-calculus problem, and it's as simple as calculating the slope of a straight line! All you need to do is find your x and y values for both points, subtract the first y value from the second, and divide it by the second x value minus the first. The equation for the average rate of change is y 2 − y 1 x 2 − x 1 {\displaystyle {\frac {y2-y1}{x2-x1}}} , or the difference in y values divided by the difference in x values. 1 Determine your x and y values. The average rate of change is the slope of a line that intersects two points on a curve. Since it's just a straight line, it's the change in y divided by the change in x.[1] If you're given the interval [x1, x2], plug each value into your equation to get y1 and y2. For example, if you had the function f(x) = x2 + 3x and the interval [1, 3], then f(1) = 12 + 3 * 1 = 1 + 3 = 4 and f(3) = 32 + 3 * 3 = 9 + 9 = 18. So your x values are 1 and 3 and your y values are 4 and 18. The variable y is also referred to as f(x) since it's the value you get when you plug an x value into the function. 2 Subtract y1 from y2 and x1 from x2. The equation for the average rate of change is y 2 − y 1 x 2 − x 1 {\displaystyle {\frac {y2-y1}{x2-x1}}} . To get your numerator, subtract y1 from y2 (AKA f(x1) from f(x2)). Then find your denominator by subtracting x1 from x2.[2] For the function f(x) = x2 + 3x over the interval [1, 3], the fraction would look like 18 − 4 3 − 1 {\displaystyle {\frac {18-4}{3-1}}} = 14 2 {\displaystyle {\frac {14}{2}}} . Advertisement 3 Divide the change in y by the change in x. Once you've set up your fraction, divide the numerator by the denominator. This gives you the average rate of change, which is also the slope of something called a secant line.[3] The average rate of change for the example function is 14 2 {\displaystyle {\frac {14}{2}}} = 7. Advertisement 1 Use the graph to find your x and y values. To find the x value of a point on a graph, draw a straight line from the point to the x-axis—you will either have to draw straight up or straight down. Where your line intersects the axis is your value. Find y by drawing a straight line to the y-axis, either to the left or right of the point.[4] If your y value is negative, draw straight up for x. If it's positive, draw straight down. If the x value is negative, draw right for y, and if it's positive draw left. 2 Determine the values for y1, y2, x1 and x2. Choose one set of points to be (x1, y1) and the other to be (x2, y2). You can choose either, but usually, people put the leftmost coordinates first (lowest value of x).[5] So if you had the points (1, 3) and (5, 10), 1 would be x1 and 3 would be y1. If you put the right coordinates first your numerator and denominator will switch from positive to negative, or from negative to positive. However, the equation gives you the same answer either way. 3 Divide the difference of the y's by the difference of the x's. Use the equation y 2 − y 1 x 2 − x 1 {\displaystyle {\frac {y2-y1}{x2-x1}}} to find the average rate of change.[6] So if you had (1, 3) and (5, 10), the slope would be 10 − 3 5 − 1 {\displaystyle {\frac {10-3}{5-1}}} = 7 4 {\displaystyle {\frac {7}{4}}} . Advertisement 1 Find your y or x values, given a certain interval. If you're asked to find the rate of change of an interval with respect to x (AKA the interval is from one x value to another), find the x values on the table. Next to the x values are their corresponding y values.[7] It's less common to be given an interval with respect to y (two y values). If you find yourself in that situation, do the same thing, but look for the given values in the y column, instead. 2 Use the equation y 2 − y 1 x 2 − x 1 {\displaystyle {\frac {y2-y1}{x2-x1}}} to find the average rate of change. Subtract the y value that corresponds with the first x value from the y value that corresponds to the second x value. Then, subtract the first x value from the second, and divide the difference in y by the difference in x.[8] Advertisement 1 Choose two points on either side of where you want to estimate. The average rate of change is the same as the slope of a secant line, or a line that goes through two points of a curve. The instant rate of change is a tangent line, which only touches a single point.[9] Choose two points that are close to where you want to find the instantaneous rate of change. 2 Calculate the average rate of change over the two points. Use the equation y 2 − y 1 x 2 − x 1 {\displaystyle {\frac {y2-y1}{x2-x1}}} to find the slope of the secant line that intersects the two points.[10] If you wanted to find the instantaneous rate of change for f(x) = x2 + 4x when x = 2, then you could make your first secant line at the points (1, 5) and (3, 21) The slope of that line is 21 − 5 3 − 1 {\displaystyle {\frac {21-5}{3-1}}} , or 8. 3 Choose two more points, both closer to the target point. Find the average rate of change over an interval even closer to where you want to find the instantaneous rate of change.[11] If you have exponents in your function you will likely need to use a calculator. Try the rate of change at one decimal place on either side of your target. For the equation (x) = x2 + 4x, you could use x1 = 1.9 and x2 = 2.1. 12.81 − 11.21 2.1 − 1.9 {\displaystyle {\frac {12.81-11.21}{2.1-1.9}}} = 1.6 .2 {\displaystyle {\frac {1.6}{.2}}} = 8. 4 Estimate the tangent line based on the secant lines. Keep finding secant lines closer and closer to the point, until you start to see your results converge on a number.[12] In the example above, a good guess would be that the instantaneous rate of change at x = 2 for f(x) = x2 + 4x is 8. Advertisement 1 Financial applications Investors analyze the speed at which stock prices change to draw conclusions about what a stock might do. If the rate of change of a stock's price is rising, for example, it means the value will go up in the short term, and if it goes down the value may decline.[13] Financial professionals also compare the rate of change of different variables to try and predict what will happen to the price of a stock. For example, if the price of a stock is rising while the rate of change is lowering, that can signal that the price is about to trend down.[14] 2 Construction and architecture Architects and engineers need precise measurements to make structural plans for buildings, bridges, and roads, including the slope of an area, or the average rate of change in elevation. 3 Sales Retailers use average rates of change in sales, prices, and margins to see what effects those variables have on profits. By comparing these businesses can see how certain products are performing. For example, if a company sees the rate of change of a low-cost product's sales go up while a high-cost product's goes down, they may lower production of the high-cost product.[15] Advertisement Ask a Question Advertisement This article was reviewed by Joseph Meyer and by wikiHow staff writer, Carmine Shannon. Joseph Meyer is a High School Math Teacher based in Pittsburgh, Pennsylvania. He is an educator at City Charter High School, where he has been teaching for over 7 years. Joseph is also the founder of Sandbox Math, an online learning community dedicated to helping students succeed in Algebra. His site is set apart by its focus on fostering genuine comprehension through step-by-step understanding (instead of just getting the correct final answer), enabling learners to identify and overcome misunderstandings and confidently take on any test they face. He received his MA in Physics from Case Western Reserve University and his BA in Physics from Baldwin Wallace University. This article has been viewed 7,592 times. Co-authors: 3 Updated: June 5, 2024 Views: 7,592 Categories: Mathematics Print Send fan mail to authors Thanks to all authors for creating a page that has been read 7,592 times. The average rate of change is defined as the average change in a function per unit over a given interval. It is the gradient of the secant line connecting the endpoints of the interval. It describes how one quantity changes with respect to another. The average rate of change is simply the gradient of the line connecting two points. For example, consider the two points 'a' and 'b' below. Point 'a' has coordinates (0, 1) and point 'b' has coordinates (4, 3). The average rate of change is simply the gradient of the red secant line shown in the diagram. We find the gradient of this line to find the average rate of change between the points. The gradient is found by dividing the change in y-coordinates by the change in x-coordinates. The change in y-coordinates between the 2 points is 2 and the change in x-coordinates between the 2 points is 4. 2 ÷ 4 = 0.5 and so, the average rate of change is 0.5. The average rate of change is a concept used in calculus to measure the gradient between two points. As the two points are brought closer together, the gradient of the secant line connecting them approaches the gradient of the tangent at the first point. In the example below, the gradient of the tangent to the curve at point 'a' is shown in green. It has a gradient of 2. The average rate of change is shown by the secant line in red. In the middle image, the gradient of the red secant line connecting points 'a' and 'b' is 0.5. In the final image, the point 'b' is brought closer to point 'a'. Here the gradient of the red secant line is now closer to the gradient of the green tangent line. As the two points get closer and closer, the gradient of the red secant line approaches the gradient of the tangent at the first point. There are two types of rate of change that can be shown on a graph: instantaneous rate of change and average rate of change. The instantaneous rate of change is equal to the gradient of the tangent to the curve at a point. This is shown by the green line in the image above. The instantaneous rate of change can be calculated by drawing a tangent at a point and calculating its gradient. Alternatively it can be calculated by differentiating the equation of the curve and then substituting in the value of x at that point. The average rate of change is equal to the gradient of the secant connecting two points. This is shown by the red line in the image above. To find the average rate of change from a graph: Find the coordinates of the two endpoints of the interval. Calculate the rise between the two points. Calculate the run between the two points. The average rate of change is equal to the rise ÷ run. For example, find the average rate of change in the interval 0 ≤ x ≤ 7 on the graph shown below. The ends of the interval have been marked by two points. Step 1. Find the coordinates of the two endpoints of the interval The first coordinate is (0, 0). The second coordinate is (7, 2) Step 2. Calculate the rise between the two points The rise is equal to the change in the y-coordinates between the two points. We go from 0 to 2 and so, the rise is equal to 2. Step 3. Calculate the run between the two points The run is equal to the change in the x-coordinates between the two points. We go from 0 to 7 and so, the run is equal to 7. Step 4. The average rate of change is equal to the rise ÷ run The rise is 2 and the run is 7. Therefore, the average rate of change is equal to 2 ÷ 7. Calculating this, this is approximately equal to 0.286. Therefore the average rate of change between (0, 0) and (7, 2) is 0.286. This means that for each 1 unit moved to the right, the graph increases by 0.286 on average in the vertical direction. The average rate of change is 0.286 and it applies to the whole region of the interval from 0 to 7. We can see in the image below that the red secant line is above the graph for almost the entire interval. Therefore it is not a good indication of the average rate of change at all points along the graph. To be a more accurate average rate of change, it would be expected that the red secant line would follow the trend of the graph more closely. The graph changes over the interval and to improve the accuracy, smaller intervals can be considered. For example, we can find the average rate of change in the interval 0 ≤ x ≤ 7. Between these points, the rise is 0.5 and the run is 5. 0.5 ÷ 5 = 0.1 and therefore the average rate of change in this interval is 0.1. This is a more accurate average rate of change for this section of the curve as the red secant line passes more closely through the middle of the curve. Now we can look at the average rate of change in the remaining section of the curve from 5 ≤ x ≤ 7. The rise is 1.5 and the run is 2. 1.5 ÷ 2 = 0.75 and so, the average rate of change in this interval is 0.75. From 0 ≤ x ≤ 5, the average rate of change is 0.1 and from 5 ≤ x ≤ 7 the average rate of change is 0.75. We can see that the graph increased much more rapidly from 5 to 7 compared to its steady rise from 0 to 5. This is why the average rate of change over the entire interval from 0 to 7 is in the middle of these values at 0.286. To find the average rate of change between two points, divide the change in y-coordinates by the change in x-coordinates over the interval. For example, find the average rate of change of in the interval . The y-coordinates of each point are found by substituting the corresponding values of x into the equation of the function. When , the y-coordinate is found by substituting into . Making the substitution, and so, the y-coordinate of this first point is equal to 1. When , the y-coordinate is found by substituting into . Making the substitution, and so, the y-coordinate of this second point is equal to 4. The average rate of change is found by dividing the change in the y-coordinates by the change in the x-coordinates. The first point has an x-coordinate of 2 and a y-coordinate of 1. The second point has an x-coordinate of 4 and a y-coordinate of 4. The change in y-coordinates is therefore 4 – 1 = 3. The change in x-coordinates is therefore 4 – 2= 2. The average rate of change is 3 ÷2 = 1.5. The average rate of change = [f(b) – f(a)]/[b – a]. This formula is used to calculate the average rate of change over the interval a≤x≤b. f(a) is the value of the y-coordinate at point a and f(b) is the value of the y-coordinate at point b. Average rate of change formula for the interval 'a' is the x-coordinate of the first pointf(a) is the y-coordinate of the first point'b' is the x-coordinate of the second pointf(b) is the y-coordinate of the second point For example between the two points (0.5, 2) and (4, 3): 'a' = 0.5f(a) = 2'b' = 4f(b) = 3 The average rate of change formula, becomes . Evaluating this formula which approximately equals 0.286. Alternate ways of writing the average rate of change formula are: To find the average rate of change from a table of values: Find the change in y-coordinate values over the interval. Find the change in x-coordinate values over the interval. Divide the change in y-coordinate values by the change in the x-coordinate values. For example, find the average rate of change in the interval from the table below. Step 1. Find the change in the y-coordinate values over the interval Over the interval from x = 3 to x = 8, the y-coordinates change from 0 to 5. Therefore the change in the y-coordinates over this interval is 5. Step 2. Find the change in x-coordinate values over the interval From x = 3 to x = 8, this is a change of 5. Step 3. Divide the change in the y-coordinate values by the change in the x-coordinate values The change in y-coordinate values is 5. The change in x-coordinate values is 5. Dividing the change in y-coordinates by the change in the x coordinates, 5 ÷ 5 = 1. Therefore the average rate of change over this interval is 1. Find the average rate of change by entering a function f(x) and an interval [a, b] in the calculator below. Step One: Evaluate the Function for A and Bf(3) = 12f(5) = 28Step Two: Use the Average Rate of Change Formulaf(b) - f(a)b - a28 - 125 - 31628The average rate of change is equal to 8 Learn how we calculated this below Add this calculator to your site Cite As: Sexton, J. (n.d.). Average Rate of Change Calculator. Inch Calculator. Retrieved May 26, 2025, from The average rate of change is an average measure of change in a function over an interval. It's the total change of the output of the function divided by the change in the function's input. To find the rate of change, first define the function that you'll use to calculate it, along with an interval [a, b]. Then, you can find the average rate of change using the slope formula. The average rate of change is actually a slope, but rather than a linear slope where the average between any two points is equal to some constant, a function is used. Average Rate of Change Formula Thus, the formula to find the average rate of change that is derived from the slope formula is: average rate of change = ΔyΔx = y2 – y1x2 – x1 = f(b) – f(a)b – a The formula states that the average rate of change is equal to the result of the function of b f(b) minus the result of the function of a f(a), divided by the value of b minus the value of a. For example, let's find the average rate of change for the interval [2, 3] using the function f(x) = x^2. Start by replacing the values in the average rate of change formula. average rate of change = 3^2 – $2^2$3 – 2 Then simplify. average rate of change = 9 – 43 – 2 average rate of change = 51 average rate of change = 5 So, in this example, the average rate of change over the interval [2, 3] is equal to 5. Frequently Asked Questions To find the average rate of change from a graph over a specified interval, simply find the coordinates of the points at each end of the interval, and use those values in the slope formula to find the average rate of change between those two points. For linear functions, the average rate of change is the same thing as the slope. However, for non-linear functions, the slope is the rate of change at a single point on the curve (and can be different at every point!), but the average rate of change is equivalent to the average slope over the specified interval. Sign Up Now &Daily Live Classes3000+ TestsStudy Material & PDFQuizzes With Detailed Analytics+ More BenefitsGet Free Access Now Welcome MathSux friends! In todays lesson, we will be going over how to find the average rate of change over an interval of a function. This may sound intimidating at first, but all we are really doing is finding the slope, yes, the one we're already familiar with over an interval given usually in the x-axis. Sounds simple enough? Check out the example below! Also, be sure to watch the video to check out multiple ways for solving these types of problems and the practice problems at the end of this post. Happy calculating! Average rate of Change over Interval Example: Step 1: The first thing we must do is figure out what this question is saying. The interval, they give us, [1,3] represents the x-values on our graph. So, they want to know what the average rate of change (or slope) is between these two x-values 1 and 3. Let's take a look and see what that means for us on the graph below! Notice we found the points on our interval [1,3]? This correlates to the coordinate points we will be working with to solve this problem (1,2) and (3,10) Also, within our interval, [1,3], we will be finding the rate of change (otherwise known as the slope) of the two coordinates (1,2) and (3,10). Check out how we are only finding a small portion of our slope from this function in the diagram below, represented by the dotted green line. Step 2: Now that we understand what we must do, find the slope within the interval [1,3], using the corresponding points, (1,2) and (3,10), all we must do is plug our numbers into the good ole' slope formula which we already know! Using our coordinates, (1,2) and (3,10), we can plug in values into the slope formula below: Try the following questions on your own on and check out the video above for more ways to answer this type of question and even more examples! Practice Questions: 1) Find the average rate of change over the interval [0,2] for the function: h(x) = 2x2 +2 2) Find the average rate of change over the interval [-4,2] for the function: g(x)=(x+3)2 3) Find the average rate of change over the interval [-2,0] for the function: f(x)=-(x+1)3 4) Find the average rate of change over the interval [0,4] for the function: h(x)=x2+2x+1 Solution: 1) 4 2) 1 3) -1 4) 6 Looking to learn more about Algebra? Check out the algebra lessons page here. Thanks for stopping by and happy calculating! Facebook ~ Twitter ~ TikTok ~ Youtube