How to use the unit circle to evaluate inverse trig functions

In other words, you know the values of $\arcsin x$/$\arctan x$/$\arccos x$ for some specific values of x. That's just fine. Inverse trigonometric functions are transcendental functions and with exceptions of a few well-known values, the result is not nicely expressible with elementary functions (you can use a calculator or any number of "approximations by hand" to get a numerical value for the angle, but that's not the same). In addition to the classical angles of multiples of 30°, which have known values of trigonometric functions, you can use the half-angle formulas and addition theorems to get other angles (inverse functions can be computed by recognizing the half-angle and angle addition expressions and reducing the calculation to a simpler expression). In fact, the angles you can construct by adding and halving of elementary angles (45° and 60°), are precisely the angles you can construct with a compass and a straightedge (constructible angles). Such constructions also mean that all the angles, for which you can get the trigonometric function without a calculator (or the inverse problem), have a nice geometric representation of how to carry out such a calculation. Construction with similar triangles and circles is thus a good idea if you have a more complicated expression that you think you can get analytically. Otherwise, there's nothing wrong about knowing a few special values. We do that all the time. Probably it's the most tricky and difficult topic throughout all High School Math.Prerequisites:Unit circleReference angle & Angle in standard positionInverse functionsRadian & degree conversionSigns of trig functionsIt asks you to solve a trig function like this:Actually it's asking you to use inverse trig function skills.Multiple solutions for inverse trig functionThe most difficult part to understand is the multiple solutions. And not all are valid.e.g. $\sin^{-1}(1/2)$ means, we know the sine value is 1/2, and we'd like to get the arc angle's measure. But there're multiple arc angles could have the very SAME sine value:Counter-clockwise: 30°, 150°Clockwise: -330°, -210°.But for making the function valid, we have to make it to 1-INPUT-1-OUTPUT.So we are to filter them out and only keep one solution, by restrict the angle's measure.The angle's measure in Sine function is restricted by DOMAIN, but in its inverse function, it became RANGE.By filtering with the RANGE, we are to get a ONLY-ONE & MUST-ONLY-ONE answer.Principle value & CalculatorSo after filtering out all other solutions, we have only ONE solution, we call it:The Principle Value of inverse trig function,which shows up in any calculator when you type it.HOW TO GET ALL POSSIBLE SOLUTIONSFirst to use a CALCULATOR to get the principle value as our base solution.Find the mirror solution of each trig function by Trig symmetry identities:sin(θ) = sin(π - θ), if θ is POSITIVE.sin(θ) = sin(-π - θ), if θ is NEGATIVE.cos(θ) = cos(-θ)tan(θ) = tan(-π + θ), if θ is POSITIVE.tan(θ) = tan(π + θ), if θ is NEGATIVE.Add periodicity to the solution to represent all periodic solutions: x = θ + 2πnUse calculator to do arcsin(0.65) and get principle value: 0.71 Rad.Apply trig identity sin(θ) = sin(π-θ) to get the mirror solution: arcsin(0.65) = π - arcsin(0.65), which'd be: 2.43 Rad.With two solutions 0.71 Rad and 2.43 Rad and add periodicity, we'll get two full solutions:x = 0.71 + 2πnx = 2.43 + 2πnUse calculator to do arcsin(-0.25) and get principle value: -0.25 Rad.Apply trig identity sin(θ) = sin(-π-θ) to get the mirror solution: arcsin(-0.25) = -π - arcsin(-0.25), which'd be: -2.89 Rad.With two solutions -0.25 Rad and 3.39 Rad and add periodicity, we'll get two full solutions:x = -0.25 + 2πnx = -2.89 + 2πnUse calculator to do arccos(-0.7) and get principle value: 2.35 Rad.Apply trig identity cos(θ) = cos(-θ) to get the mirror solution: arccos(-0.7) = - arcsin(-0.7), which'd be: -2.35 Rad.With two solutions 2.35 Rad and -2.35 Rad and add periodicity, we'll get two full solutions:x = 2.35 + 2πnx = -2.35 + 2πnUse calculator to do arccos(0.4) and get principle value: 1.16 Rad.Apply trig identity cos(θ) = cos(-θ) to get the mirror solution: arccos(0.4) = - arcsin(0.4), which'd be: -1.16 Rad.With two solutions 1.16 Rad and -1.16 Rad and add periodicity, we'll get two full solutions:x = 1.16 + 2πnx = -1.16 + 2πnThe range differs for each arc-function:Arcsin(x)=θ: -90° < θ < 90°, means angle only exists in Q.1 and Q.4Arccos(x)=θ: 0° < θ < 180°, means angle only exists in Q.1 and Q.2Arctan(x)=θ: -90° < θ < 90°, means angle only exists in Q.1 and Q.4Tricks:About the signs like 90 and -90, just to think about the CLOCKWISE and COUNTER-CLOCKWISE.Special Value of trig function solvingWith special trig values, we really don't need calculator at all, but only to look at the picture of Complete Unit Circle. Or not even that if you can remember it.Notice: The Complete Unit Circle only shows counter-clockwise angle measures which means ONLY POSITIVE ANGLES, so you have to do your own math to get the NEGATIVE ANGLES, aka. clockwise angle measures.Refer to youtube: Evaluating Inverse Trigonometric Functions, Basic Introduction, Examples & Practice Problems: $\sin^{-1}(1/2)$, $\sin^{-1}(\sqrt{3}/2)$, [[$\sin^{-1}(-1/2)$]]($\sin^{-1}(-\sqrt{3}/2)$, $\sin^{-1}(0)$, $\sin^{-1}(1)$, $\sin^{-1}(-1)$, $\cos^{-1}(1/2)$, $\cos^{-1}(-\sqrt{3}/2)$, $\cos^{-1}(-\sqrt{2}/2)$, $\cos^{-1}(0)$, $\tan^{-1}(0)$, $\tan^{-1}(1)$, $\tan^{-1}(-1)$, $\tan^{-1}(\sqrt{3})$, $\tan^{-1}(-\sqrt{3}/3)$, review all.Example: Solve $\sin^{-1}(1/2)$By looking at the unit circle, we know there're multiple arc measures could get a sine value 1/2, which are 30°, 150°, -210°, -330°.Filter out all others by the Range [-90°, 90°], we will get 30° is the ONLY-ONE and MUST-ONLY-ONE answer.Example: Solve $\sin^{-1}(-1/2)$Look at the unit circle, we know the answer are 210°, 330°, -30°, -150°With filtering by range [-90°, 90°], so -30° is the only answer.Example: Solve $\cos^{-1}(\sqrt{2}/2)$Basic trig equationsOnce you figure out how to solve the original functions solution, it's so easy with this basic one.Step-by-step solutions are as below:Simplify the equation to sin(θ)=?? form.Solve θ with TWO solutions in the form of θ = ?? + 2πn.Replace θ with the expression of x and solve the equation for x.Khan practice.Example: Solve 20sin(10x) − 10 = 5Simplify the equation and get sin(θ) = 3/4Solve sin(θ) = 3/4 get solutions for is: θ = 0.85 + 2πn and θ = 2.29 + 2πnReplace θ with expression of x, which make 10x = 0.85 + 2πn and 10x = 2.29 + 2πnSolve equations for x, get x = 0.085 + 0.2πn and x = 0.229 + 0.2πnComposition of Inverse trig functions Explain why and how the domains of sine, cosine, and tangent must be restricted to create an inverse function. Use the restricted domains of the sine, cosine, and tangent, and reason to reason about the domains and ranges of the inverse functions. Evaluate inverse trig expressions and equations Quick Lesson Plan With questions 1 and 2, we'd like students to understand that the inverse sine is not a function because the original sine function has repeated outputs, meaning it is not one-to-one. In order to make the inverse a function, they need to restrict the domain of the original, which becomes the range of the inverse. While the conventional range of the inverse function is [-(π/2), (π/2)], the students can also choose [(π/2), (3π/2)] if they'd like. We want them to be able to reason with the domains and ranges of the inverse, so giving them the freedom to choose their interval will help to make the connections. Formalize Later For the debrief, make sure you explain how the restricted domains they chose in #4, #6, and #8 are the actual ranges of the inverse functions. If they chose a different interval than the conventional one, make sure you explain why we choose[-(π/2), (π/2)] instead of [(π/2), (3π/2)] for y = arcsin(x). I usually connect it back to the calculator and how it can only provide one answer, which is always between -90 and 90 degrees (for arcsin). Note to students that though this is the convention, it is a rather arbitrary choice, just as some notation is a choice made by the mathematical community that everyone agrees to adhere to. They should also see the notation for inverse as arcsin, arccos, and arctan in addition to the usual "-1" superscript. Most inverse trig evaluating comes from the Unit Circle, so show the connection from the graphs of sine, cosine, and tangent to the quadrants of the Unit Circle in the Important Ideas. Preview Preview Page 2 Something went wrong. Wait a moment and try again. This is the second part of my discussion on using inverse trig functions. In this video, I go over examples of how we can use the unit circle to solve problems involving inverse trig functions and special angles. This is a worksheet that will be worked on in class that has some practice problems and review from the videos in section 6.6.